UNIT-IV

PUSH DOWN AUTOMATA

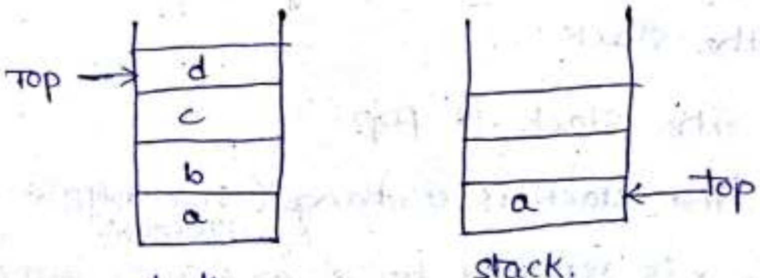
* Introduction * Basic model [formal definition] * Graphical notation
* Instantaneous Description (ID) * Acceptance of PDA.

Introduction:-

* A PDA is a way to implement a CFG in a similar way we can design FA for Regular Grammar.
* PDA is more powerful than finite state machine.
* FSM has a very limited memory. But a PDA has more memory.
* $\boxed{PDA = FSM + stack}$
* A stack is a way we arrange elements one on the top of stack.
* A stack does two basic operations.
  - i) PUSH :- A new element is added at the top of the stack.
  - ii) pop :- The top element of the stack is read and remove.

Ex:- push(a)
push(b)
push(c)
push(d)

pop()
pop()
pop()



stack   stack.

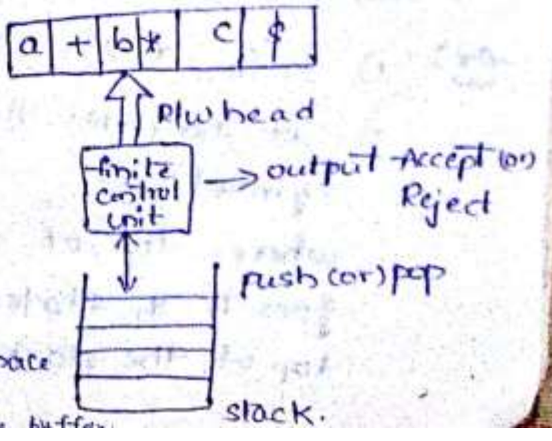
* **Basic model of PDA:-**

PDA has three Components.

i) input tape

ii) finite Control unit.

iii) stack



* A stack with infinite size.
* It has unlimited amount of storage space
* Used to store data and remove the data temporarily which is read by FSU from input tape buffer

Formal definition:-

- Mathematically a PDA is defined with 7 tuples like
  $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where

  $Q \rightarrow$ finite and non-Empty set of states

  $\Sigma \rightarrow$ finite and non-Empty set of input symbols

  $\Gamma \rightarrow$ finite and non-Empty set of stack symbols.

  $\delta \rightarrow$ It is a transition function which is defined as

  δ:

  $$\boxed{Q \times \{\Sigma \cup \epsilon\} \times \Gamma^* \longrightarrow Q \times \Gamma^*}$$

  $$\boxed{Q \times \Sigma^* \times \Gamma^* \longrightarrow Q \times \Gamma^*}$$

where 'δ' takes three tuples as i/p like $\delta(q, a, x)$

where i) q is a state in Q.

ii) a is either an i/p symbol in Σ (or) a is also belongs ϵ.

iii) x is a stack symbol i.e; member of Γ

iv) The o/p of δ is finite set of pairs like (p, γ)

where, p: It is a new state.

γ: It is a set of stack symbols. that replace 'x' at the top of the stack.

Ex:- 1) If $\gamma = \epsilon$ then the stack is pop.

2) If $\gamma = x$ then the stack is unchange (since bypass operation)

3) If $\gamma = yz$ then x is replaced by z and 'y' is pushed on to the stack.

Ex:- 1) $\delta(q_0, a, z) = (q_1, yz)$

$\Rightarrow$ It indicates that from state q_0, reading i/p symbol 'a'.

where, top of the stack z. then the finite control goes to q_1 state and adding the element 'y' to the top of the stack.

2) $\delta(q_1, a, z) = (q_2, \epsilon)$

⇒ It indicates that 'z' is removed from the stack and state is changed from q_1 to q_2.

3) $\delta(q_1, a, z) = (q_2, z)$

⇒ It indicates that on reading symbol 'a' state is changing from q_1 to q_2 and there is no change in the stack (bypass operation)

q_0 → It is the initial state.

$\boxed{q_0 \in Q}$

z_0 → It is the start stack symbol.

$\boxed{z_0 \in \Gamma}$

F → It is the set of final (or) accepting state and ($F \subseteq Q$).

Graphical representation :-

The Graphical representation of PDA is Transition diagram

FA :-

$A \xrightarrow{a} B$

$\delta(A, a) = B$

PDA :-

$A \xrightarrow{a,b \to c} B$ (next state)

$\delta(A, a, b) = (B, C)$

Current state

i/p symbols may be 'ε'.

Symbol in top of the stack

This symbol is poped.

ε means the stack neither read nor poped.

This symbols is pushed onto stack

may be ε

the symbol is pushed.

Instantaneous description :-

It is used to describe the configuration of PDA at given Instance.

ID remembers the state and stack content.

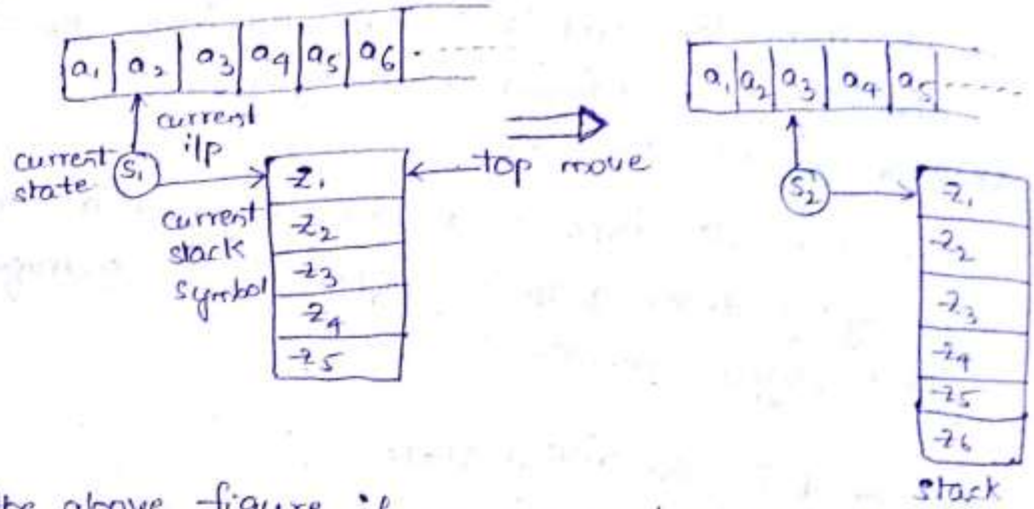
It was defined by Triple(q, w, γ) where

q → is a state.

w → input symbols of string

γ → is a string of stack symbols.

Example :- $\delta(P, a, z_0) = (q, \beta)$



From the above figure, if we are reading the current i/p symbol 'a_2' at current state 's_1' and current stack symbol 'z_1' then after a move we will reach to state s_2. and there will be some new symbol on the top of the stack. This description can be represented as.

1) push operation :-

$\delta(q_0, a, z_0) = (q_1, az_0)$ push 'a' onto the stack.

q_0: current state (or) present state

a: current i/p symbol

z_0: current stack top symbol.

q_1: change the state from q_0 to q_1

2) pop operation :-

$\delta(q_0, x, y) = (q_1, \epsilon)$

q_0: current state

x: current i/p symbol

y: current stack top symbol.

q_1: change the state from q_0 to q_1

ϵ: pop the stack (or) removing the stack.

Acceptance of PDA :-

There are two ways to accept a language by PDA. They are i) Accepted by empty stack.

ii) Accepted by final state.

Accepted by empty stack :-

The given language Accepted by empty stack to be defined as $L(N) = \{ w \mid \delta(q_0, w, z_0) \overset{*}{\Rightarrow} (P, \epsilon, \epsilon) \text{ for some } p \text{ in } Q \}$

that is, if stack becomes empty After scanning entire string then it is accepted by PDA. otherwise, not accepted.

Accepted by final string:-

The given language accepted by final state to be defined as

$L(M) = \{ w \mid \delta(q_0, w, z_0) \overset{*}{\Rightarrow} (p, \epsilon, \gamma)$ for some $p \in F$ and $\gamma \in \Gamma^* \}$

that is, eventhough stack is not empty, after scanning i/p string. if the finite control reaches to the final state then it is accepted. otherwise, not accepted.

Design of PDA:-

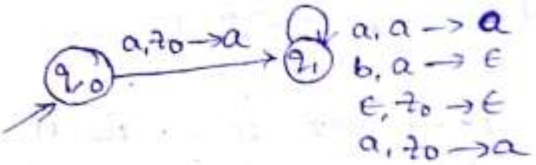
Types of PDA:-

i) Deterministic PDA :- if all derivations in the design has to give only single move

ii) Non Deterministic PDA :- if derivation generates more than one move in the designing of a particular task.

1) Design a PDA that accepts equal no. of A's and B's.

sol:-



δ: $\delta(q_0, a, z_0) = (q_1, a z_0)$

$\delta(q_1, a, a) = (q_1, aa)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$\delta(q_1, a, z_0) = (q_1, a z_0)$

∴ The PDA machine for the above language is defined as

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ where $Q = \{q_0, q_1\}$

$\Sigma = \{a, b, \epsilon\}$

$\Gamma = \{z_0\}$

δ :

$q_0 = \{q_0\}$

$z_0 = \{z_0\}$

$F = \{ \}$

(1) Read A's → push operation, Read B's → push operation

Consider a String $w = \{abab\}$ Read a's

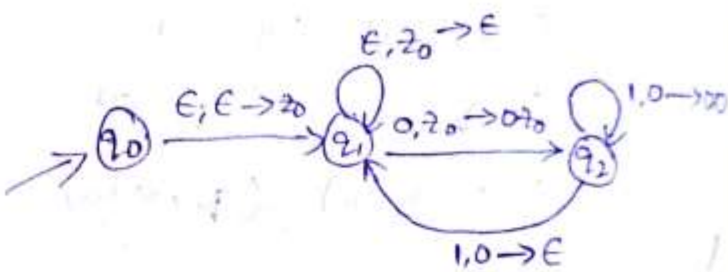
$\delta(q_0, abab, z_0) = \delta(q_1, bab, a z_0)$

③ Design a PDA for the language $L = \{0^n \text{ and } 1^{2n} \mid n \geq 1\}$

Sol:- $L = \{0^n 1^{2n} \mid n \geq 1\}$

Read one 0 $\longrightarrow$ push

Read two 1's $\longrightarrow$ pop



$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$

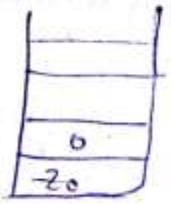
$\delta(q_1, 0, z_0) = (q_2, 0z_0)$

$\delta(q_2, 0, 0) = (q_2, 00)$

$\delta(q_2, 1, 0) = (q_2, 00)$

$\delta(q_2, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon, \epsilon)$



$\delta(q_1, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, 1, 0) = (q_1, 0z_0)$

④ consider the string $w = \{001111\}$

$\delta(q_1, 001111, z_0) = \delta(q_2, 01111, 0z_0)$

$= \delta(q_2, 1111, 00)$

$= \delta(q_2, 111, 00)$

$= \delta(q_1, 11, 0z_0)$

$= \delta(q_1, 1, 0z_0)$

stack

$= \delta(q_1, \epsilon, z_0)$

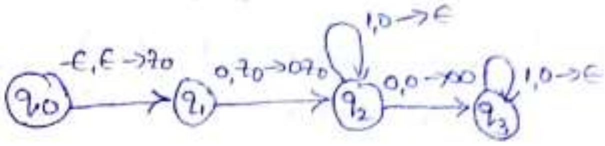
$= \delta(q_1, \epsilon, \epsilon)$

//.

Design a PDA for the language $L = \{0^n 1^n \mid n \geq 1\}$

Read 0's → PUSH

Read 1's → pop



$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$

$\delta(q_1, 0, z_0) = (q_2, 0z_0)$

$\delta(q_2, 0, 0) = (q_2, 00)$

$\delta(q_2, 1, 0) = (q_3, \epsilon)$

$\delta(q_3, 1, 0) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, z_0) = (q_3, \epsilon, \epsilon)$

* Design a PDA for the language $L = \{ww^R \mid w \in (a+b)^*\}$

ii) $L = \{wcw^R \mid w \in (a+b)^*\}$

sol) i) $L = \{ww^R \mid w \in (a+b)^*\}$

In this language contains palindrome string. i.e;

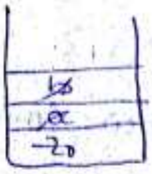
if $w = ab$, $w^R = ba$ then $ww^R = abba$ is a palindrome.

* We can read no. of a's and b's and pushed them into stack until we can reach the midposition of i/p string.

* In the mid position we can't read any i/p and can't push onto stack.

* After midposition when we read a (or) b then pop them from the stack. This process is repeated until stack is empty.

$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$



$\delta(q_1, a, z_0) = (q_1, az_0)$

$\delta(q_1, b, z_0) = (q_1, bz_0)$

$\delta(q_1, \epsilon, \epsilon) = (q_2, z_0)$

$\delta(q_2, a, a) = (q_3, \epsilon)$

$\delta(q_2, b, b) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, z_0) = (q_4, \epsilon, \epsilon)$

ii) $L = \{ wcw^R \mid w = (a+b)^* \}$

$w = ab$

$w^R = ba$

$wcw^R = abcba$

$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$

$\delta(q_1, a, z) = (q_1, az_0)$

$\delta(q_1, b, z_0) = (q_1, bz_0)$

$\delta(q_1, a, a) = (q_1, aa)$

$\delta(q_1, a, b) = (q_1, ab)$

$\delta(q_1, b, a) = (q_1, ba)$

$\delta(q_1, b, b) = (q_1, bb)$

$\delta(q_1, c, z_0) = (q_2, z_0)$

$\delta(q_1, c, a) = (q_2, a)$

$\delta(q_1, c, b) = (q_2, b)$

$\delta(q_2, a, a) = (q_3, \epsilon)$

$\delta(q_3, b, b) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, z_0) = (q_4, \epsilon, \epsilon)$

Deterministic PUSHDOWN Automata :-

- A DPDA is 7 Tuple like $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

where Q is finite and non empty set of states

Σ is finite and nonempty set of i/p Alphabet

Γ is finite set of stack symbols

δ is a mapping function used for mapping (or) moving from current state to next state. is defined as $\delta(q_0, x, z_0) = (q, x\beta)$ where

q_0 is current state

x is current i/p symbol

z_0 is current stack symbol

q is next state

$x\beta$ shows top of the stack

if δ denotes a unique transition for each i/p then PDA is said to be deterministic PDA

Ex:- 1) $L = \{ a^n b^n \mid n \geq 1 \}$

2) $L = \{ wcw^R \mid w = (a+b)^* \}$

Nondeterministic PDA :-

It is a 7 tuple like $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ where

Q is finite and non Empty set of states

Σ is finite and non Empty set of i/p Alphabet

Γ is finite set of stack symbols.

δ is a mapping function used for moving from current state to next state and is defined as $\delta(q_0, x, z_0) = (q, x\beta)$

q_0 is current state
x is current i/p symbol
z_0 is stack symbol
q is next state
$x\beta$ is top of the stack.

if 'δ' denotes more than one transition for a particular i/p symbol then the PDA is said to be non-deterministic PDA.

Ex:- $L = \{ ww^R \mid w = (a+b)^* \}$

Context free grammar and push down automata:-

Conversion of CFG to PDA.
conversion of PDA to CFG.

i) Conversion of CFG to PDA:-

* For constructing a PDA from given CFG it is necessary to convert this CFG to some Normal form like GNF.
* For converting given CFG to PDA, By this method the necessary condition is that the first symbol on RHS of production rule must be a terminal symbol. This rule that can be used to obtain PDA from CFG.

Algorithm:-

Rule 1:- for non-terminal symbols, add following rule

$\delta(q, \epsilon, A) = (q, \alpha)$ where the production rule is $A \to \alpha$.

Rule 2:- for each terminal symbols, add following rule

$\delta(q, a, a) = (q, \epsilon)$ for every terminal symbol 'a' in given CFG.

Ex:- construct a PDA for the given CFG
$S \to 0BB$
$B \to 0S$
$B \to 1S$
$B \to 0$

Sol:- The given CFG $G = (V, T, P, S)$ where V = non-terminals $\{S, B\}$

$T = \{0, 1\}$

$P:$ $S \rightarrow 0BB$
$B \rightarrow 0S$
$B \rightarrow 1S$
$B \rightarrow 0$

$S = \{S\}$

Rule 1 :- $A \rightarrow \alpha$
$\delta(q, \epsilon, A) = (q, \alpha)$

$S \rightarrow 0BB$
$\delta(q, \epsilon, S) = (q, 0BB)$

$B \rightarrow 0S$
$\delta(q, \epsilon, B) = (q, 0S)$

$B \rightarrow 1S$
$\delta(q, \epsilon, B) = (q, 1S)$

$B \rightarrow 0$
$\delta(q, \epsilon, B) = (q, 0)$

Rule :- 2

Terminals
$\delta(q, a, a) = (q, \epsilon)$
$T = \{0, 1\}$
$\delta(q, 0, 0) = (q, \epsilon)$
$\delta(q, 1, 1) = (q, \epsilon)$

$\therefore$ The corresponding PDA for the given CFG is defined as

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q\}$
$\Sigma = \{0, 1\}$
$\Gamma = \{S, B, 0, 1\}$
δ = it is a transition symbol defined as
$\delta(q, \epsilon, S) = (q, 0BB)$
$\delta(q, \epsilon, B) = (q, 0S)$
$\delta(q, \epsilon, B) = (q, 1S)$
$\delta(q, \epsilon, B) = (q, 0)$
$\delta(q, 0, 0) = (q, \epsilon)$
$\delta(q, 1, 1) = (q, \epsilon)$

$q_0 = \{q\}$
$z_0 = \{z_0\}$
$F = \{\ \}$

2) construct a PDA for the following CFG $S \to 0S1$
$S \to A$
$A \to 1A0 \mid S \mid \epsilon$.

sol:- The Given CFG is $S \to 0S1$
$S \to A$
$A \to 1A0$
$A \to S$
$A \to \epsilon$.

elimination of ϵ-production:-

$A \to \epsilon$ ×
$S \to A$
$S \to \epsilon$ ×
$S \to 0S1$
$S \to 0\epsilon1$
$S \to 01$

$A \to 1A0$
$A \to 1\epsilon0$
$A \to 10$
$A \to S$
$A \to \epsilon$ ×

$\therefore$ $S \to 0S1 \mid 01$
$S \to A$
$A \to 1A0 \mid 10$
$A \to S$.

elimination of unit productions:-

$S \to A$ ×
$S \to 1A0 \mid 10$

$A \to S$ ×
$A \to 0S1 \mid 01$

$\therefore$ The resultant CFG is.

$S \to 1A0$
$S \to 10$
$A \to 0S1$
$A \to 01$

$\therefore$ The Simplified CFG is.

$S \to 1A0$
$S \to 10$
$A \to 0S1 \mid 1A0 \mid 10$
$A \to 01$
$S \to 0S1 \mid 01$

Method-2
$P \to 1$
$Q \to 0$

$S \to 1A0$
$S \to 1AQ$ ✓

$S \to 10$
$S \to 1Q$ ✓

$A \to 0S1$
$A \to 0SP$ ✓

$A \to 10$
$A \to 1Q$ ✓

$S \to 0S1$
$S \to 0SP$ ✓

$A \to 1A0$
$A \to 1AQ$ ✓

$A \to 01$
$A \to 0P$ ✓

$S \to 01$
$S \to 0P$. ✓

$\therefore$ The simplified CFG in GNF is
$S \to 1AQ$ $A \to 0SP$
$S \to 1Q$ $A \to 1Q$
$S \to 0SP$ $A \to 1AQ$
$S \to 0P$ $A \to 0P$
$P \to 1$
$Q \to 0$

Rule-1
$A \to \alpha$.

$\therefore$ The PDA is

$S \to 1AQ$.
$\delta(q, \epsilon, S) = (q, 1AQ)$

$S \to 0SP$
$\delta(q, \epsilon, S) = (q, 0SP)$

$A \to 0SP$
$\delta(q, \epsilon, A) = (q, 0SP)$

$S \to 1Q$
$\delta(q, \epsilon, S) = (q, 1Q)$

$S \to 0P$
$\delta(q, \epsilon, S) = (q, 0P)$

$A \to 1Q$
$\delta(q, \epsilon, A) = (q, 1Q)$

$A \to 1AQ$

$\delta(q, \epsilon, A) = (q, 1AQ)$

$A \to 0P$

$\delta(q, \epsilon, A) = (q, 0P)$

$P \to 1$

$\delta(q, \epsilon, P) = (q, 1)$

$Q \to 0$

$\delta(q, \epsilon, Q) = (q, 0)$

Rule 2:– Terminals

$\delta(q, a, a) = (q, \epsilon)$

$T = \{0, 1\}$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

method:2

The Given CFG is $S \to 0S1$

$S \to A$

$A \to 1A0$

$A \to S$

$A \to \epsilon$.

The resultant PDA is $S \to 0S1$

$\delta(q, \epsilon, S) = (q, 0S1)$

$S \to A$

$\delta(q, \epsilon, S) = (q, A)$

$A \to 1A0$

$\delta(q, \epsilon, A) = (q, 1A0)$

$A \to S$

$\delta(q, \epsilon, A) = (q, S)$

$A \to \epsilon$

$\delta(q, \epsilon, A) = (q, \epsilon)$

construct PDA for the following CFG $S \to aABB \mid aAA$

$A \to aBB \mid a$

$B \to bBB \mid A$

sol:- The Given CFG is $S \to aABB$

$S \to aAA$

$A \to aBB$

$A \to a$

$B \to bBB$

$B \to A$

elimination of unit production:–

$B \to A$ ×

$B \to aBB$

$B \to a$.

∴ After eliminating unit production $B \to A$, The resultant CFG in GNF is $S \to aABB$

$S \to aAA$

$A \to aBB$

$A \to a$

$B \to bBB$

$B \to aBB$

$B \to a$

The PDA is:

$S \to aABB$

$\delta(q, \epsilon, S) = (q, aABB)$

$S \to aAA$

$\delta(q, \epsilon, S) = (q, aAA)$

$A \to aBB$

$\delta(q, \epsilon, A) = (q, aBB)$

$A \to a$

$\delta(q, \epsilon, A) = (q, a)$

$B \to bBB$

$\delta(q, \epsilon, B) = (q, bBB)$

$B \to aBB$

$\delta(q, \epsilon, B) = (q, aBB)$

$B \to a$

$\delta(q, \epsilon, B) = (q, a)$

Conversion of PDA to CFG :—

* If $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is a PDA. then their exists CFG 'G' which is accepted by PDA (M).

Let 'G' be a CFG which is generated by PDA. The 'G' can be defined as $G = (V, T, P, S)$ where 'S' is the start symbol and the set of non-terminals $V = \{S, q, q', z_0\}$ where $S, q, q' \in Q$ and $z_0 \in \Gamma$.

Now, we get set of production rules using the following algorithm.

Algorithm :—

Rule 1 :— The start symbol production rule can be $S \to [q, z_0, q']$

where q indicates present state

q' indicates next state.

z_0 is the stack symbol.

Rule 2 :- If there exists a move of PDA as then the production rule can be return as $\delta(q, a, z_0) = (q', \epsilon)$

$[q, z_0, q'] \to a$

3) If there exists a move of PDA as $\delta(q, a, Z_0) = (q', Z_1 Z_2 Z_3 \dots Z_n)$ then the production rules can be written as

$$[q, Z_0, q'] \rightarrow a[q, Z_1, q_1][q_1, Z_2, q_2][q_2, Z_3, q_3] \dots [q_{n-1}, Z_n, q']$$

Ex:- construct a CFG from the following PDA $M = (\{q_0, q_1\}, \{0, 1\}, \{S, A\}, \delta, q_0, S, \phi)$ and

δ:

$\delta(q_0, 1, S) = (q_0, AS)$

$\delta(q_0, \epsilon, S) = (q_0, \epsilon)$

$\delta(q_0, 1, A) = (q_0, AA)$

$\delta(q_0, 0, A) = (q_1, A)$

$\delta(q_1, 1, A) = (q_1, \epsilon)$

$\delta(q_1, 0, S) = (q_0, S)$

Sol:- Let we will construct a CFG $G = (V, T, P, S)$ where

$T = \{0, 1\}$

$V = \{S \cup [q_0, S, q_0], [q_0, S, q_1], [q_1, S, q_0], [q_1, S, q_1], [q_0, A, q_0], [q_0, A, q_1], [q_1, A, q_0], [q_1, A, q_1]\}$

Now, Let us build the production rules as.

using rule ① the production rules for start symbol is

$P_1: S \rightarrow [q_0, S, q_0]$

$P_2: S \rightarrow [q_0, S, q_1]$

using Rule ③ of the Algorithm for the $\delta(q_0, 1, S) = (q_0, AS)$

$q_0 < {q_0 \atop q_1}$ $P_3: [q_0, S, q_0] \rightarrow 1[q_0, A, q_0][q_0, S, q_0]$

$q_0 < {q_0 \atop q_1}$ $P_4: [q_0, S, q_0] \rightarrow 1[q_0, A, q_1][q_1, S, q_0]$

$P_5: [q_0, S, q_1] \rightarrow 1[q_0, A, q_0][q_0, S, q_1]$

$P_6: [q_0, S, q_1] \rightarrow 1[q_0, A, q_1][q_1, S, q_1]$

Now, for $\delta(q_0, \epsilon, S) = (q_0, \epsilon)$ using Rule ② of Algorithm we get.

$P_7: [q_0, S, q_0] \rightarrow \epsilon$

P_8: ~~$[q_0, \cancel{\$}, A] \rightarrow q_0, A$~~

Now for $\delta(q_0, 1, A) = (q_0, AA)$ using Rule ③ of Algorithm

$P_8: [q_0, A, q_0] \rightarrow 1[q_0, A, q_0][q_0, A, q_0]$

Now for $\delta(q_0, 0, A) = (q_1, A)$ using Rule ⑤ of Algorithm

$P_9: [q_0, A, q_0] \rightarrow 1 [q_0, A, q_1] [q_1, A, q_0]$

$P_{10}: [q_0, A, q_1] \rightarrow 1 [q_0, A, q_0] [q_0, A, q_1]$

$P_{11}: [q_0, A, q_1] \rightarrow 1 [q_0, A, q_1] [q_1, A, q_1]$

Now, for $\delta(q_0, 0, A) = (q_1, A)$ using

$q_0 < \begin{matrix} q_0 \\ q_1 \end{matrix}$ $\quad q_1 < \begin{matrix} q_0 \\ q_1 \end{matrix}$

$P_{12}: [q_0, A, q_0] \longrightarrow 0 [q_1, A, q_0]$

$P_{13}: [q_0, A, q_1] \longrightarrow 0 [q_1, A, q_1]$

Now, for $\delta(q_1, 1, A) = (q_1, \epsilon)$

$P_{14}: [q_1, A, q_1] \rightarrow 1$

Now, for $\delta(q_1, 0, S) = (q_0, S)$

$q_1 < \begin{matrix} q_0 \\ q_1 \end{matrix}$ $\quad q_0 < \begin{matrix} q_0 \\ q_1 \end{matrix}$

$P_{15}: [q_1, S, q_0] \rightarrow 0 [q_0, S, q_0]$

$P_{16}: [q_1, S, q_1] \rightarrow 0 [q_0, S, q_1]$

PDA with two stacks: —

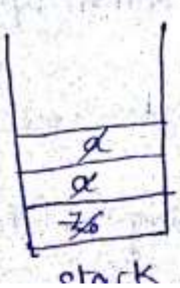
ⓐ **PDA with one stack:**

① $L = \{a^n b^n \mid n \geq 1\}$

consider the string $w = aabb$

read a's → push

read b's → pop



stack

a a b b $

when stack is empty then the string aabb is accepted.

① $L = \{a^n b^n c^n \mid n \geq 1\}$

consider string $w = aabbcc$

read a's $\rightarrow$ PUSH

read b's $\rightarrow$ pop

read c's $\rightarrow$ nochange

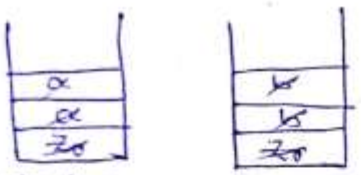
stack.

aabbcc\$

when read c there is no change in stack without completion of reading string 'w' the stack is empty. so, string is not accepted.

ⓑ PDA with two stacks :—

$L = \{a^n b^n c^n \mid n \geq 1\}$



stack 1 stack 2

$w =$ aabbcc\$

read a's $\rightarrow$ PUSH (on $stack_1$)

read b's $\rightarrow$ PUSH (on $stack_2$)

read c's $\rightarrow$ pop (a from $stack_1$ and 'b' from $stack_2$)

when two stacks are empty then string 'w' is accepted.

$\therefore$ The PDA with two stacks is more powerful than a PDA with one stack.

FA + 0-stack = NFA or DFA

FA + 1-stack = PDA.

FA + 2-stack = PDA with two stacks.

- Applications of PDA:—

* uced for deriving a string from the grammar.
* used for designing Top-down parser and bottom-up parser in compiler design.
* It works on regular grammar and Context-free grammars
* It accepts regular Language and CFL.
* It has remembering capability by maintaining a stack.
* It is more powerful than FA.